

## EXPLANATION OF AN OBSERVATION IN THE SIZE QUANTIZATION EFFECT

V.M. KENKRE

*Institute for Fundamental Studies, The University of Rochester,  
Department of Physics and Astronomy, Rochester, New York 14627, USA*

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It is shown that the explanation of a peculiar observed fact concerning the temperature dependence of transport parameters for size quantized films lies in an analysis carried out by Chaplik in which it is proved that the two-dimensional character of the electron motion leads to the inapplicability of the Born approximation to the scattering problem.

When boundaries in real space are imposed on electrons or other excitations forcing them to reside within finite limits, discrete quantum levels appear in their spectrum. This phenomenon can be observed in extremely thin ( $< 500 \text{ \AA}$ ) films of semiconducting or semimetallic substances like bismuth and is known as the quantum size effect (QSO) [1].

The QSO has been the subject of many publications in recent times. Two papers, one reporting theoretical work [2] and the other describing experimental observations [3] have just appeared in the literature. It is obvious that the authors were not aware of each other's work and the purpose of this letter is to show how an observed fact described in ref. [3] can be explained in terms of the analysis in ref. [2].

Garcia et al. [3] have reported, observing a peculiar temperature dependence of several transport parameters in size quantized films. Specifically, these are the resistivity, the Hall coefficient and the magneto-resistance. The first two exhibit a temperature maximum and the last possesses a strange kink in its temperature plot. The authors of ref. [3] have not provided a satisfactory explanation and have stated that the problem is open to investigation.

In ref. [2] Chaplik has carried out a theoretical analysis of some questions pertaining to the motion of electrons in thin films. As can be easily shown [1, 4] the boundaries of a thin film cause a splitting of the 3-dimensional momentum (or quasi-momentum) space into separate 2-dimensional subspaces and consequently force a certain 2-dimensional character on the motion of the electrons. Peculiarities possessed by 2-dimensional motion must therefore make their presence

felt in the properties of size quantized films. Chaplik has shown [2] that one of these is the inapplicability of the Born approximation to the problem of impurity scattering.\* He has then proceeded to resume the entire Born series and has obtained various expressions for transport quantities of interest. These differ significantly from the ones one arrives at in the Born approximation and this is connected with the occurrence of localized states.

It is possible to show [6] that this analysis leads to a maximum in the temperature dependence of the resistivity. The explanation of the results of Garcia et al. appears therefore to be contained in Chaplik's analysis. In fact, Garcia et al. have reported that the temperature maximum shifts towards lower temperatures on increasing the film thickness [3, fig. 10] and we shall show here that even this qualitative behaviour is already contained in Chaplik's results. Denoting the temperature maximum in the resistivity as  $T_m$ , we can obtain from eq. (6) of ref. [6].

$$T_m = (c_1/a^2) \exp(-ac_2) , \quad (1)$$

where  $a$  is the film thickness and  $c_1$  and  $c_2$  are positive constants. Eq. (1) gives

$$\partial T_m / \partial a = -(c_2 + 2/a) T_m . \quad (2)$$

Noting that  $c_2$ ,  $a$  and  $T_m$  are all positive we conclude that eq. (2) indeed explains the qualitative behaviour of the shift in the temperature maximum.

\* Another is the absence of Bose-Einstein condensation in a gas of excitons [5].

Eq. (2) can also be put to a (semi)quantitative test since the  $c_2$  in it is given simply by

$$c_2 = 1/2|f| \quad (3)$$

where  $|f|$  is the absolute value of the scattering amplitude of a zero-energy electron at an impurity atom in a bulk sample. Also arguments originating in the results of refs. [2] and [6], and analogous to the preceding ones can be used in the explanation of the peculiar temperature dependence (the kink in the magneto-resistance and the maximum in the Hall coefficient) of the other transport parameters. These considerations will be reported elsewhere [5].

We thus propose that it is indeed the localized states of Chaplik and the breakdown of the Born approximation (arising out of the 2-dimensional

electron motion) that gives rise to the peculiar temperature dependence reported by Garcia, Kao and Strongin.

#### References

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