

## Effects of dissipation on nonlinearity in transport: Evolution and integrability properties in a molecular dimer

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The effects of dissipation on the transport of quasiparticles obeying the nonlinear discrete Schrödinger equation are studied. Dissipation is introduced via the standard stochastic Liouville equation and, for a molecular dimer, a closed integro-differential equation is derived for the time dependence of the probability difference at the two molecular sites. The equation is solved numerically in the general case and analytically in a certain limit involving weak nonlinearity and damping, and the solutions are used to describe the scattering spectrum. The integrability of the equations of motion is investigated and it is shown that the equations of motion do not possess the Painlevé property.

### I. INTRODUCTION AND THE INTEGRO-DIFFERENTIAL EQUATION

In the present paper we address the effects of the interplay of dissipation with transport nonlinearity. The former will be introduced via the standard stochastic Liouville equation in the manner shown below. The latter is characterized by the nonlinear discrete Schrödinger equation introduced by Davydov<sup>1</sup> in his studies of quasiparticle motion in biological systems, and has received a great deal of attention in recent times.<sup>2-8</sup> It has the form

$$i \frac{dc_m}{dt} = E_m c_m + \sum_n V_{mn} c_n - \chi |c_m|^2 c_m, \quad (1.1)$$

where  $c_m$  is the probability amplitude for a given excitation to be found at the site  $m$  of a crystal or an aggregate,  $E_m$  denotes the local energy at site  $m$ ,  $V_{mn}$  is the intersite matrix element between sites  $m$  and  $n$ , and  $\chi$  is a quantity that is directly related to the coupling of the excitation with the vibrational modes of the system. When  $\chi=0$ , Eq. (1.1) reduces to the usual Schrödinger equation expanded in the Wannier basis under the tight-binding approximation. When  $\chi \neq 0$  the nonlinear term introduces an effective lowering of the local energy at site  $m$ . This has the effect of altering the coherent features of the quasiparticle motion in a drastic fashion, e.g., it can lead to the self-trapping of the quasiparticle.

It is straightforward to show<sup>3,5</sup> that Eq. (1.1) results in the following Liouville-von Neumann equation for the density matrix  $\rho$ :

$$i \frac{d\rho_{mn}}{dt} = (E_m - E_n) \rho_{mn} + \sum_r (V_{mr} \rho_{rn} - \rho_{mr} V_{rn}) - \chi (\rho_{mm} - \rho_{nn}) \rho_{mn}, \quad (1.2)$$

where  $\rho_{mn}$  are the elements of the density matrix in the site basis. The standard procedure<sup>9-13</sup> of introducing dissipation into the Liouville-von Neumann equation is to include terms which cause a decay of the off-diagonal elements of the density matrix in a suitable representation. If we apply that procedure to Eq. (1.2) we obtain

$$i \frac{d\rho_{mn}}{dt} = (E_m - E_n) \rho_{mn} + \sum_r (V_{mr} \rho_{rn} - \rho_{mr} V_{rn}) - \chi (\rho_{mm} - \rho_{nn}) \rho_{mn} - i\alpha (1 - \delta_{mn}) \rho_{mn}, \quad (1.3)$$

where  $\alpha$  describes the rate of destruction of the off-diagonal elements of the density matrix. It is also possible to look upon  $\alpha$  as the scattering rate of a quasimomentum state in the band of the quasiparticle.

We now consider a degenerate molecular dimer, i.e., a system in which the quasiparticle moves between two sites with identical energies which can be set to zero without any loss of generality. We take  $m, n=1, 2$  and  $V_{12} = V_{21} = V$  and obtain the following set of equations:

$$i \frac{d}{dt} \rho_{11} = -V(\rho_{12} - \rho_{21}), \quad (1.4a)$$

$$i \frac{d}{dt} \rho_{22} = V(\rho_{12} - \rho_{21}), \quad (1.4b)$$

$$i \frac{d}{dt} \rho_{12} = -V(\rho_{11} - \rho_{22}) - \chi(\rho_{11} - \rho_{22})\rho_{12} - i\alpha\rho_{12}, \quad (1.4c)$$

$$i \frac{d}{dt} \rho_{21} = V(\rho_{11} - \rho_{22}) + \chi(\rho_{11} - \rho_{22})\rho_{21} - i\alpha\rho_{21}. \quad (1.4d)$$

Our aim is to derive from Eq. (1.4) a single equation for the evolution of the difference in the occupation probabilities of the two sites. We, therefore, define a new set of variables  $p, q, r, s$  which are linear combinations of the old variables and equal to  $\rho_{11} - \rho_{22}, \rho_{12} - \rho_{21}$ ,

$\rho_{12} + \rho_{21}, \rho_{11} + \rho_{22}$ , respectively. The system of Eqs. (1.4) can then be written as

$$\frac{d}{dt}p = i2Vq, \quad (1.5a)$$

$$\frac{d}{dt}q = i2Vp + i\chi pr - \alpha q, \quad (1.5b)$$

$$\frac{d}{dt}r = i\chi pq - \alpha r, \quad (1.5c)$$

$$\frac{d}{dt}s = 0, \quad (1.5d)$$

and reduced further through the elimination of  $q$  to yield

$$\frac{d^2p}{dt^2} + \alpha \frac{dp}{dt} + 4V^2p = -2V\chi pr, \quad (1.6a)$$

$$\frac{d}{dt}(e^{\alpha t}r) = \frac{\chi}{4V}e^{\alpha t} \frac{d}{dt}p^2. \quad (1.6b)$$

Finally, by solving Eq. (1.6b) formally, and substituting into Eq. (1.6a) we arrive at the following equation for  $p(t)$ :

$$\begin{aligned} \frac{d^2p}{dt^2} + \alpha \frac{dp}{dt} + \left[ 4V^2 + 2V\chi r_0 e^{-\alpha t} - \frac{\chi^2}{2} p_0 e^{-\alpha t} \right] p + \frac{\chi^2}{2} p^3 \\ = \frac{\chi^2 \alpha}{2} p \int_0^t dt' e^{-\alpha(t-t')} p^2(t'), \end{aligned} \quad (1.7)$$

where  $p_0, r_0$  are the initial values of  $p, r$ , respectively.

Equation (1.7) is the point of departure for our analysis in what follows. It is a closed, nonlinear, integro-differential equation for  $p(t)$ , the difference in the occupation probabilities of the two sites of our dimer system. To the extent that our use of the stochastic Liouville equation as a procedure to introduce dissipation is a valid one, Eq. (1.7) describes the dynamics of the dimer in the simultaneous presence of nonlinearity and damping. We observe that when  $\alpha = \chi = 0$  the equation for a harmonic oscillator results, whereas for  $\alpha \neq 0$  and  $\chi = 0$  we get an equation for a damped harmonic (linear) oscillator. Finally, for  $\chi \neq 0$  and  $\alpha = 0$ , we obtain the equation for the nonlinear degenerate dimer, first studied by Kenkre and Campbell.<sup>3</sup>

The rest of the paper is divided as follows. In Sec. II Eq. (1.7) is studied analytically as well as numerically. Analytic solutions are possible only if some simplifying assumptions concerning the system parameters are made. When these assumptions are not valid, numerical analysis allows us to study the effects of the interplay between damping and nonlinearity in the transfer of the particle moving between the two sites. Such motion may be probed experimentally through scattering observations involving probe particles such as neutrons. In Sec. III we, therefore, evaluate the scattering spectrum of the nonlinear dimer with damping and comment on its relation to spectra of both the linear dimer with dissipation as well as that of the nonlinear dimer without dissipation.

During the last ten years, several authors<sup>14-17</sup> have noted an apparent connection between integrable, nonlinear evolution equations and Painlevé property. This, therefore, seemed to be an appropriate tool for studying

the dimer equations. This was particularly so since they can be solved by Jacobi elliptic functions when one of the parameters vanishes. (The Jacobi elliptic functions may be considered as degenerate cases of the Painlevé transcendents,<sup>18</sup> which certainly have the Painlevé property.) Therefore, in Sec. IV we will perform a Painlevé analysis of our equations for the dimer case, following the approach of Ref. 14. Unfortunately, we will show that the equations are not of the Painlevé type unless the aforementioned parameter vanishes, in which case the general solution is already known.<sup>3,6</sup>

## II. ANALYTIC AND NUMERICAL SOLUTIONS

It is possible to write Eq. (1.7) as a nonlinear differential equation of the third order.<sup>19</sup> Here, however, we will directly use Eq. (1.7) and first show that it can be solved through an approximate but analytical procedure for a certain range of the parameters  $\chi$  and  $\alpha$ . Let us assume that  $\chi^2 \alpha$  is small enough so that the integral term on the right-hand side of the equation does not contribute significantly in the time evolution. In addition, if  $\alpha$  is chosen to be small, the exponential terms on the left-hand side of Eq. (1.7) can be set equal to unity, i.e.,  $e^{-\alpha t} \approx 1$ , a condition clearly valid for times shorter than or on the order of  $1/\alpha$ . With these constraints on the parameters  $\chi$  and  $\alpha$ , Eq. (1.7) reduces to

$$\frac{d^2p}{dt^2} + \alpha \frac{dp}{dt} = Ap - Bp^3, \quad (2.1a)$$

$$A = \frac{\chi^2}{2} p_0 - 4V^2 - 2\chi V r_0, \quad (2.1b)$$

$$B = \frac{\chi^2}{2}. \quad (2.1c)$$

If  $p(t)$  is interpreted as the position of a classical particle at time  $t$ , Eq. (2.1) then represents the time evolution for the particle, constrained to move in a symmetric double-well potential, under the simultaneous action of an external dissipative force proportional to the particle velocity.

Equation (2.1) can be integrated exactly for a certain range of values of  $\chi$  and  $\alpha$ . We can eliminate the term involving the first derivative of  $p(t)$  through a simultaneous transformation of both the dependent and independent variables  $p$  and  $t$  to  $y$  and  $z$ , respectively:

$$p = -\gamma zy \quad \text{and} \quad (2.2)$$

$$\frac{dz}{dt} = -\gamma z,$$

where  $\gamma$  is an arbitrary parameter which remains to be determined. The transformation given in Eq. (2.2) is equivalent to

$$p = e^{-\gamma t} \quad \text{and} \quad (2.3)$$

$$z = -\frac{1}{\gamma} e^{-\gamma t}.$$

Equation (2.1a) can be written equivalently in terms of the new set of variables  $y$  and  $z$ :

$$\gamma^2 z^2 \frac{d^2 y}{dz^2} - (\alpha - 3\gamma)\gamma z \frac{dy}{dz} = (A + \alpha\gamma - \gamma^2)y - B\gamma^2 z^2 y^3. \quad (2.4)$$

The choice

$$\gamma = \frac{\alpha}{3} \quad (2.5)$$

eliminates the first-derivative term in Eq. (2.4). If, in addition, we introduce the constraint

$$\frac{\alpha^2}{9} + \frac{A}{2} = 0, \quad (2.6)$$

the differential equation (2.4) then reduces to

$$\frac{d^2 y}{dz^2} + By^3 = 0. \quad (2.7)$$

This last equation can be solved using standard methods from the theory of elliptic integrals.<sup>20,21</sup> We obtain

$$p(t) = \left[ \frac{\chi}{2C} \right]^{-1/2} e^{-(\alpha/3)t} \times \text{cn} \left[ \frac{3}{\alpha} (\chi C)^{1/2} (e^{-(\alpha/3)t} - 1) + \phi_0 \left| \frac{1}{\sqrt{2}} \right. \right], \quad (2.8a)$$

$$\phi_0 = \text{cn}^{-1} \left[ p_0 \left[ \frac{\chi}{2C} \right]^{1/2} \left| \frac{1}{\sqrt{2}} \right. \right], \quad (2.8b)$$

$$C^2 = 2V^2 p_0^2 \left[ 1 + 2 \left[ \frac{\chi}{4V} \right] r_0 \right], \quad (2.8c)$$

$$\left[ \frac{\alpha}{2V} \right]^2 = \frac{9}{2} \left[ 1 + 2 \left[ \frac{\chi}{4V} \right] r_0 - 2 \left[ \frac{\chi}{4V} \right]^2 p_0^2 \right]. \quad (2.8d)$$

In Eq. (2.8)  $\text{cn}(u | k)$  denotes the Jacobian elliptic cosine of argument  $u$  and modulus  $k$  with  $\text{cn}^{-1}(u | k)$  denoting the inverse of that function.

We note that in deriving the solution given by Eq. (2.8) the simplifying assumption was made, with no loss of generality, that the off-diagonal matrix elements of the initial density matrix are real; this is equivalent to taking  $(dp/dt)_{t=0} = 0$ . Using properties of elliptic functions,<sup>20,21</sup> we can show that the solution given in Eq. (2.8) satisfies this condition. We should also point out that the exact solution (2.8) is valid only for those values of  $\alpha$  and  $\chi$  which satisfy the constraint of Eq. (2.8d), which also depends on  $V$ . If we fix the value of the latter, i.e., assume that the value of the resonant interaction is given, we can rewrite Eq. (2.8d) (when  $r_0 = 0$  and  $p_0 = 1$ ) as

$$(\alpha/2V)^2 = \frac{9}{2} \left[ 1 - 2 \left[ \frac{\chi}{4V} \right]^2 \right]. \quad (2.9)$$

For real  $\alpha$ , the necessary condition for which Eq. (2.9) is true, i.e.,  $\chi/4V < \sqrt{2}/2$ , is satisfied since Eq. (2.9) is valid for small values of  $\chi$ .

We now turn to the numerical solutions of the equations governing the evolution of the probability difference. Figure 1 shows a comparison between the analytic solution (2.8) of the approximate equation (2.1) and the numerical solution of the original equation (1.7) for several values of  $\alpha/2V$  and  $\chi/4V$ . We observe reasonably good agreement between the two solutions. In Fig. 2 we present the numerical solution of the original equation (1.7) for various values of nonlinearity and damping. In Fig. 2(a) we fix the amount of dissipation ( $\alpha/2V = 0.1$ ) and plot the probability difference for various values of the nonlinearity parameter. The initial conditions chosen correspond to placing the particle initially in one of the sites of the dimer, i.e.,  $p_0 = 1$  and  $r_0 = 0$ .

Figure 2(a) shows the tendency of the dimer to exhibit the self-trapping transition.<sup>3,5,6</sup> This is indicated by the fact that, provided the nonlinearity parameter is large enough, the initial oscillations of  $p(t)$  are not symmetric about the origin. However, dissipation causes the system to reach a symmetric steady state for long times. The probability is equalized over the two sites, irrespective of the initial conditions, i.e., as  $t \rightarrow \infty$ ,  $p(t) \rightarrow 0$ . While Fig. 2(a) shows the effects of varying the amount of nonlinearity for a given amount of damping, Figs. 2(b) and 2(c) show the effect of varying the degree of damping. The "free" region (cn evolution) is represented in Fig. 2(b) and the "self-trapping" region (dn evolution) in Fig. 2(c). In all cases we obtain the expected behavior: the damping agency forces the asymptotic equalization of the probabilities of the two sites. As is well known,<sup>10,11,13</sup> this behavior is expected since the presence of a single decay rate  $\alpha$  for all elements of the density matrix in the stochastic Liouville equation make the latter only a high-temperature instrument.

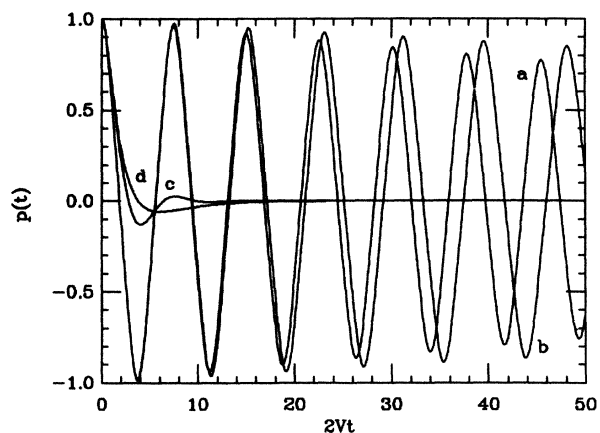


FIG. 1. Probability difference  $p(t)$  plotted as a function of  $2Vt$ . In curves  $a$  and  $b$  the values of  $\alpha/2V$  and  $\chi/4V$  are equal to 0.01 and 0.707, respectively, whereas in curves  $c$  and  $d$  the corresponding values are 0.9 and 0.640. Curves  $a$  and  $c$  represent the numerical solution of Eq. (1.7), whereas  $b$  and  $d$  represent the exact solution of the approximate equation (2.1a). For initial conditions we chose  $p_0 = 1$  and  $r_0 = 0$ ; the values of  $\alpha$  and  $\chi$  used satisfy the constraint given by Eq. (2.8d).

### III. CALCULATIONS OF THE SPECTRUM OF THE SCATTERING FUNCTION

In a scattering experiment probe particles such as neutrons exchange momentum  $q$  and energy  $\omega$  (in units of  $\hbar$ ) with the moving particles. The observable of interest, the quasielastic scattering function  $S(q, \omega)$ , can be expressed

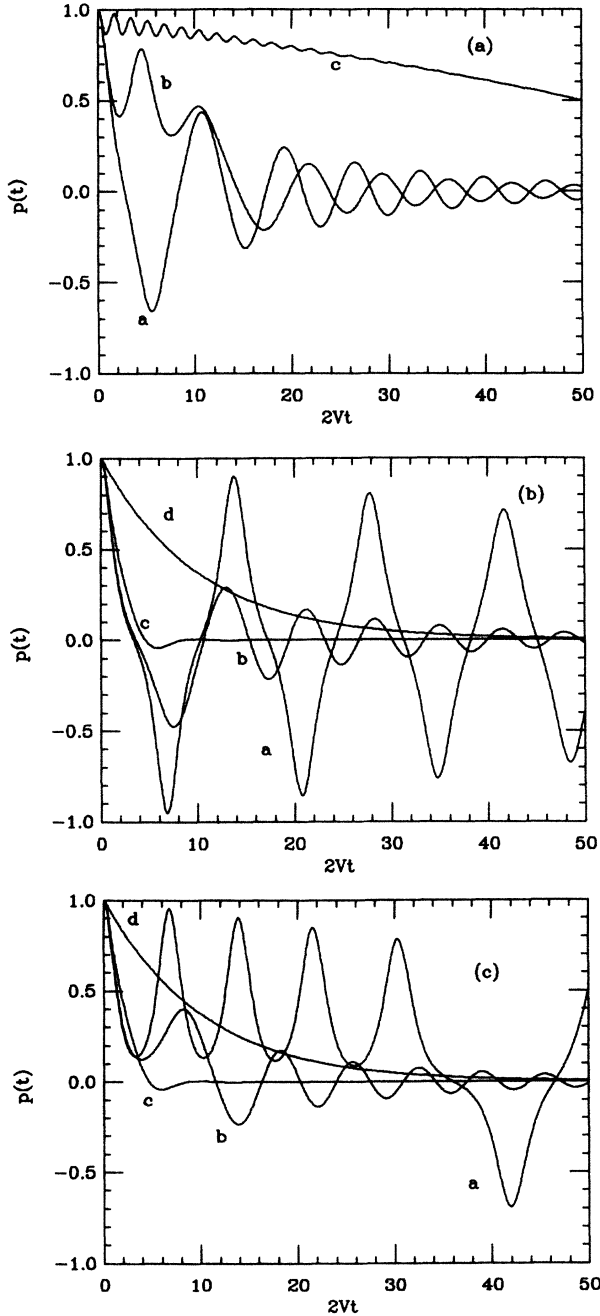


FIG. 2. Numerical solution of Eq. (1.7) presented for the probability difference  $p(t)$  as a function of time for several values of nonlinearity and damping. In (a)  $\alpha/2V=0.1$  and  $\chi/4V$  takes the values 0.95 (curve a), 1.1 (curve b), and 2.0 (curve c). In (b)  $\chi/4V=0.99$  and  $\alpha/4V$  takes the values 0.01 (curve a), 0.1 (curve b), 1.0 (curve c), and 10.0 (curve d). In (c) we have  $\chi/4V=1.01$  and  $\alpha/2V$  assumes the values 0.01 (curve a), 0.1 (curve b), 1.0 (curve c), and 10.0 (curve d).

in terms of the self-correlation function  $I(q, \omega)$  through the relations<sup>22,23</sup>

$$S(q, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt e^{-i\omega t} I(q, t), \quad (3.1)$$

$$I(q, t) = (\text{Tre}^{-\beta H})^{-1} \text{Tre}^{-\beta H} e^{iqx} e^{iH} e^{-iqx} e^{-iH}. \quad (3.2)$$

At high temperatures, the intermediate function  $I(q, t)$  reduces to the probability propagator for the motion of the particle from site 0 at  $t=0$  to a site  $m$  at time  $t$ .<sup>13,24,25</sup> For a dimer, this propagator is nothing but  $p(t)$ , i.e., the difference in the probability of occupation of the two sites of the dimer.<sup>13,4</sup> In this case, the quasielastic part of the scattering function  $S(\omega)$  containing all the essential information on the scattering process is given by

$$S(\omega) = S(\pi, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt e^{-i\omega t} p(t), \quad (3.3)$$

where  $p(t)$  is equal to the difference of the probabilities of occupation of the two sites of the dimer.

The neutron scattering spectrum for the nonlinear dimer in the absence of damping has been analyzed in Ref. 4. It was shown there that for finite nonlinearity  $\chi$  the line shape consists of an infinite series of  $\delta$  functions before and after the self-trapping transition while, at the transition, they collapse into a continuous curve which has the functional form of a hyperbolic secant. Also, on increasing the nonlinearity, the discrete spectrum changes in a way reminiscent of the behavior of the linear damped dimer, which exhibits line broadening and motional narrowing.

For the damped nonlinear dimer under study in the present paper we present analytic and numerical results for the scattering function in correspondence with the analytic solutions of the approximate form and the numerical solutions of the exact form of the  $p(t)$  equation, which we have carried out in Sec. II. A Fourier expansion of the cn function in terms of trigonometric cosines leads from the approximate equation (2.8) to

$$p(t) = \left[ \frac{\chi}{C} \right]^{-1/2} \frac{1}{K} \sum_{m=0}^{\infty} \text{sech} \left[ (2m+1) \frac{\pi}{2} \right] I_m, \quad (3.4a)$$

$$I_m = \int_0^{\infty} dt \cos(\omega t) e^{-(\alpha/3)t} \cos \left[ (2m+1) \frac{\pi}{2} \frac{u}{K} \right], \quad (3.4b)$$

$$u = \frac{3}{\alpha} (\chi C)^{1/2} (e^{-(\alpha/3)t} - 1) + \phi_0, \quad (3.4c)$$

where  $K \equiv K(1/\sqrt{2})$  is the complete elliptic integral of the first kind with modulus  $1/\sqrt{2}$ . Equation (3.4) differs from the equation for the scattering spectrum of the nonlinear dimer given earlier<sup>4</sup> in the absence of damping in that, for the latter case, the integrals with  $I_m$  in Eq. (3.4b) are  $\delta$  functions. For small damping, the discrete  $\delta$  functions broaden into Lorentzians.

To analyze the scattering function for arbitrary values of  $\chi$  and  $\alpha$  we carry out numerical calculations of  $S(\omega)$ . Figure 3(a) shows the scattering function  $S(\omega)$  as a function of the frequency  $\omega$  for small values of nonlinearity, whereas Fig. 3(b) shows  $S(\omega)$  for large values of non-

linearity at which the undamped dimer would be close to the self-trapping transition (but still in the free region). The signature of the nonlinearity, viz., the multiplicity of the scattering peaks,<sup>4</sup> is visible for small values of damping. This disappears, though, when large values of damping broaden the peaks, causing them to lose their identity and to coalesce into a central line at  $\omega=0$ . The self-trapping region is exhibited in Figs. 3(c) and 3(d). Thus the features observed are similar to those of the undamped nonlinear dimer<sup>4</sup> except for the broadening (absence of  $\delta$  functions).

#### IV. PAINLEVÉ ANALYSIS

The Painlevé analysis of a nonlinear differential equation (or a system) is associated with the study of the singularity content of their solutions. A differential equation is said to possess the Painlevé property, or to be of  $P$ -type, if its solutions, viewed as functions of a complex variable, contain no movable singularities other than poles, i.e., if it has no movable branch points or essential singularities. A critical point is termed movable if its location depends on the initial conditions chosen; thus, if different initial conditions are employed, the singularity in the solution “moves” in the complex plane. The possi-

ble connection between the Painlevé property and the “integrability” of a nonlinear system has been repeatedly emphasized by Ablowitz, Ramani, and Segur,<sup>14</sup> whose method we follow.

We first rewrite the coordinates and parameters of system (1.5) in terms of dimensionless variables, introducing the new quantities  $x_1=p$ ,  $x_2=iq$ ,  $x_3=r$ ,  $\tau=2Vt$ , where the variable  $\tau$  will be assumed to take on complex values. [These variables cause the initial density matrix to be parametrized in the form  $\rho=\frac{1}{2}(I+\mathbf{P}\cdot\boldsymbol{\sigma})$ , with  $(x_3, x_2, x_1)$  as  $\mathbf{P}$  and  $s$  being taken as 1.] The system then takes the form

$$\frac{d}{d\tau}x_1=x_2, \quad (4.1a)$$

$$\frac{d}{d\tau}x_2=-x_1-\alpha'x_2-\chi'x_1x_3, \quad (4.1b)$$

$$\frac{d}{d\tau}x_3=-\alpha'x_3+\chi'x_1x_2, \quad (4.1c)$$

where  $\alpha'=\alpha/2V$  and  $\chi'=\chi/2V$ . We will now follow the algorithm of Ablowitz, Ramani, and Segur<sup>14</sup> that provides sufficient conditions for a given equation or system to be of  $P$ -type. One assumes that the solutions can be

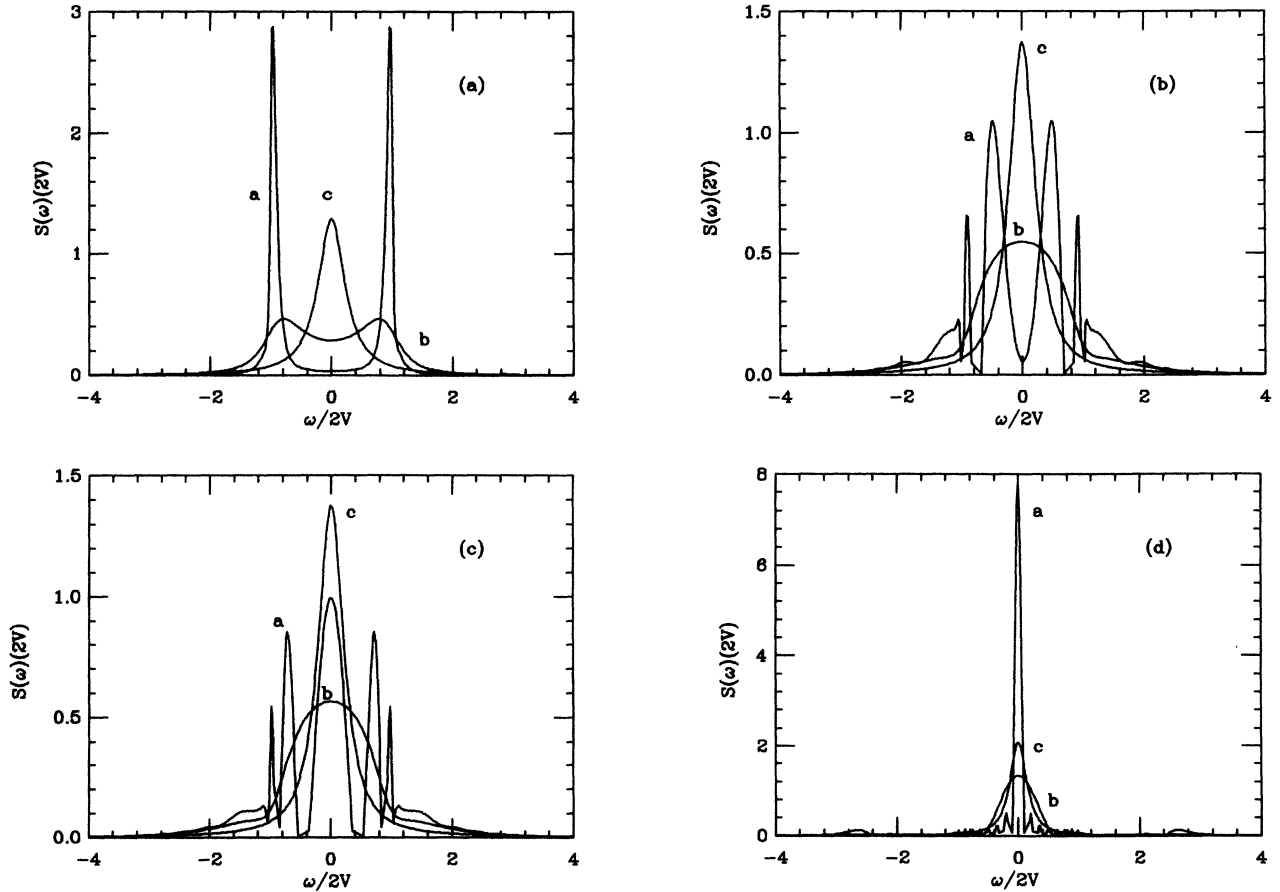


FIG. 3. Scattering function  $S(\omega)$  presented as a function of  $\omega$ . In (a)  $\chi/4V=0.4$  and  $\alpha/2V$  takes the values 0.1 (curve a), 0.8 (curve b), and 4.0 (curve c). In (b)  $\chi/4V=0.99$  and in (c) it is 1.01 for the same values of  $\alpha/2V$  as before. Finally, in (d)  $\chi/4V=1.6$  with  $\alpha/4V$  assuming the values 0.1 (curve a), 0.8 (curve b), and 6.0 (curve c).

expanded around some possible singularity  $\tau_0$ , and first looks at the behavior of only the leading terms. Therefore suppose that

$$x_i = \delta_i (\tau - \tau_0)^{p_i} + \dots, \quad i = 1, 2, 3. \quad (4.2)$$

One easily obtains the following values:

$$p_1 = -1, \quad p_2 = -2 = p_3,$$

with

$$\delta_1 = \mp i\mu = -\delta_2, \quad \delta_3 = -\mu, \quad (4.3)$$

where  $\mu \equiv 2/\chi'$ . The fact that the exponents in the leading terms of the solution to Eqs. (4.1) are integers signifies that the singularities that will be subsequently obtained are not algebraic branch points. The next step in the investigation of the singularity structure requires that one determine the "resonances" of the solution. Looking for the next terms in the expansion of the solution we may write

$$\begin{aligned} x_1 &= \mp i\mu z^{-1} + \zeta_1 z^{s-1} + \dots, \\ x_2 &= \pm i\mu z^{-2} + \zeta_2 z^{s-2} + \dots, \\ x_3 &= -\mu z^{-2} + \zeta_3 z^{s-2} + \dots, \end{aligned} \quad (4.4)$$

where  $z = \tau - \tau_0$ . To leading order in the  $\zeta_i$ , insertion of these forms into the original system gives a homogeneous algebraic system of three equations in three unknowns. Vanishing of the determinant requires that

$$(s-2)(s-4)(s+1) = 0. \quad (4.5)$$

The purpose of this procedure is twofold. First, the integer values of the solutions tells us that any branch points to be expected in the solution of the system are not algebraic. The other is more subtle. Granted the validity of a (Laurent) series expansion of the type already begun, substitution of a complete series expansion into the system will determine recursion relations that determine explicitly all the other solutions, with the exception of (at least) one coefficient at those powers which (in the language of Ablowitz, Ramani, and Segur) correspond to resonances. The undetermined coefficients (at a power corresponding to a positive, integral solution of the "resonance" equation) allow the constants of integration of the system to be evidenced. (In a simple Taylor series expansion of the solution, as in the more usual series expansions of the solutions of differential equations, the constant simply appear as the first coefficient in the series.) On the other hand, the root  $s = -1$  simply corresponds to the arbitrariness of the choice of the point  $\tau_0$ .

Having determined the integral values of the resonance powers, one would simply proceed to determine recursion relations for all the rest of the coefficients. However, there is one more undesirable possibility: there might still be logarithmic branch points in the solutions. The existence of logarithmic branch points will be signaled by the inconsistency of the equations to determine the constants of integration. Therefore, the last part of the procedure is to substitute, into the system of differential equations, expansions of the solutions that are complete

through the powers corresponding to all the solutions of the resonance equation:

$$\begin{aligned} x_1 &= \mp i\mu z^{-1} + a_1 + a_2 z + a_3 z^2 + a_4 z^3 + \dots, \\ x_2 &= \pm i\mu z^{-2} + b_1 z^{-1} + b_2 + b_3 z + b_4 z^2 + \dots, \\ x_3 &= -i\mu z^{-2} + c_1 z^{-1} + c_2 + c_3 z + c_4 z^2 + \dots. \end{aligned} \quad (4.6)$$

If there are no logarithmic branch points in the solutions of the system, substitution of (4.6) into (4.1) should determine the coefficients  $a_i, b_i, c_i, i = 1, 2, \dots$ , term by term, and thus evaluate the series representation of the solution of the system around any point in the complex  $\tau$  plane (provided that  $\tau_0$  is not an essential singularity). During the process two arbitrary constants will be obtained. Together with  $\tau_0$  these are the "constants of integration," which are to be determined by the initial conditions of the problem. Unfortunately, this particular part of the analysis fails for our system of equations. Sequential solution of the equations generated by (4.6) gives reasonable solutions through the third step ( $s=2$ ), with one arbitrary constant generated at  $s=2$ . At the value  $s=4$ , the solutions indicated by (4.6) are inconsistent, requiring that  $\alpha=0$  in order for their continuation. To indicate this fact explicitly, we note that an expansion consistent through  $s=2$  requires the addition, at that point, of logarithmic terms in the expansion, giving the following result (where  $c_2$  and  $a_4$  are the arbitrary constants, one determined at each of the resonances):

$$\begin{aligned} x_1 &= \mp i\mu z^{-1} \pm i\alpha'/\chi' \mp i(1/\chi' + c_2)z \\ &\quad \mp i(\alpha'/2\chi')z^2 + z^3[a_4 \pm 2i(\alpha'^2/5\chi')\ln(z)] + \dots, \\ x_2 &= \pm i\mu z^{-2} \mp i(1/\chi' + c_2) \mp i(\alpha'/\chi')z \\ &\quad + z^2[3a_4 \pm 6i(\alpha'^2/5\chi')\ln(z)] + \dots, \\ x_3 &= -\mu z^{-2} + c_2 + z^2[\mp 2ia_4 + \alpha'^2/2\chi' - 1/2\chi' \\ &\quad - c_2 - \chi'c_2^2/2 \\ &\quad + 4(\alpha'^2/5\chi')\ln(z)] + \dots. \end{aligned} \quad (4.7)$$

The presence of a logarithmic branch point in the solutions indicates that the system (4.1) [or, equivalently, (1.5)] does not possess the Painlevé property except for the very special case of  $\alpha=0$ , i.e., in the complete absence of damping, in which case the system is already known to be integrable.<sup>3</sup>

## V. CONCLUSIONS

The purpose of this paper is to study the effects of the interplay of dissipation and nonlinearity in transport. The specific system we analyze is a dimer; the formalism we used to introduce dissipation is that of the well-known stochastic Liouville equation,<sup>9-13</sup> and the equation we used to describe nonlinearity is the discrete nonlinear Schrödinger equation.

Our results show that the system has a tendency towards self-trapping, as in the undamped nonlinear dimer, but that it reaches an equilibrium distribution for long

times with equal probability on the two sites of the dimer, no matter what the initial condition. The scattering spectrum of the system is seen to be similar to that in the absence of dissipation but shows the additional feature of broadening. For large damping, the nonlinear features are hidden by those contributed by dissipation.

Finally, we obtained a series solution for the equations of motion of the nonlinear dimer and showed that they do *not* possess the Painlevé property.

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