Department of Physics and Astronomy, University of New Mexico

Quantum Mechanics Preliminary Examination

Fall 2011

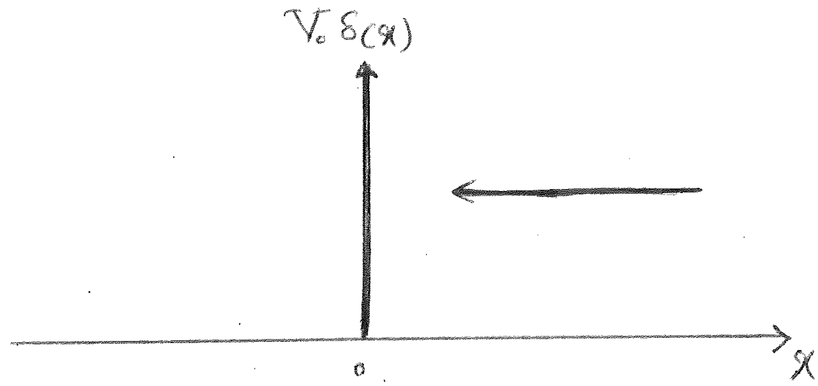
Instructions:

- The exam consists of 10 short-answer problems (10 points each).
- Partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed.
- Total time: 3 hours.

1– An electron microscope uses a beam of electrons to produce a magnified image of the specimen. Find the minimum kinetic energy of electrons so that the microscope has enough resolution to observe micron size objects.

2– A free particle of mass m is arranged in a Gaussian wavepacket with initial values $\langle X \rangle_0 = 0$, $\Delta X_0 = a$, and $\langle P \rangle_0 = p_0$ at $t = 0$. What is ΔP_0? Find $\langle X \rangle(t)$, $\langle P \rangle(t)$, and $\Delta P(t)$. Describe in words the behavior of $\Delta X(t)$.

3– A particle with mass m and energy E moves in one dimension and approaches a localized potential barrier $V = V_0\delta(x)$ from the right. Find the particle wavefunction (up to an overall constant) and the transmission probability.



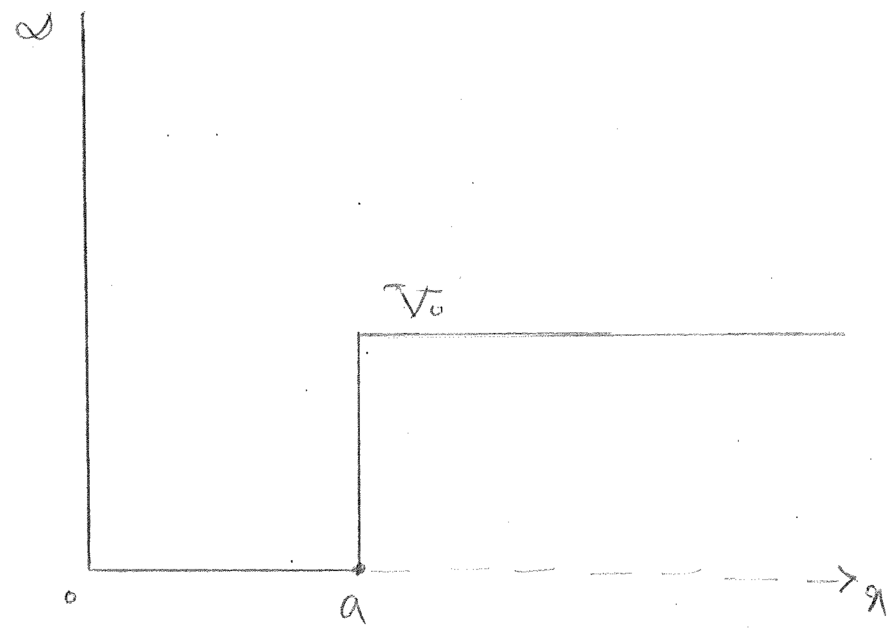
4– A simple harmonic oscillator with mass m and frequency ω is in its ground state. The frequency is suddenly changed to ω'. Find the average energy of the oscillator right after the change in frequency.

5– The Hamiltonian for a system consisting of two nonidentical spin-1/2 particles is given by

$$H = a\vec{S}_1 \cdot \vec{S}_2 + b(S_{1z} + S_{2z})\,.$$

The first term represents spin-spin interaction between the particles, and the second term comes from the interaction of particles with an external magnetic field in the z direction. Considering only the spin degrees of freedom, find the energy eigenstates and their corresponding eigenvalues for this system.

6– A particle of mass m moves in an asymmetric one-dimensional potential well. The wall at the origin is infinitely high, and the potential height of the wall at $x = a$ is V_0. Does this system always have a ground state? Explain.



7– Consider a simple harmonic oscillator in one dimension with mass m and frequency ω. A perturbation of the form λx^4 is added to the potential. Find corrections to the energy and wavefunction of the ground state to first order in perturbation theory.

8– Consider two spin-1/2 particles freely moving in three dimensions. In the center-of-mass frame, the wavefunction of this system can be written as the product of an orbital angular momentum part and a spin part. From the addition of angular momentum we know that one can obtain total angular momentum $J = 1$ for $L = 0$ & $S = 1$, $L = 1$ & $S = 0$, $L = 1$ & $S = 1$. Which of these are acceptable if the particles are identical? Explain.

9– An electron initially in the eigenstate of $\hat{S}_y$ with eigenvalue $\hbar/2$ is placed in a uniform magnetic field $B = B_0\hat{z}$. What is the probability that the electron is found in the eigenstate of $\hat{S}_y$ with eigenvalue $-\hbar/2$ at a later time t?

10– Consider the $n = 3$ energy level of the Hydrogen atom. What is the total number of states at this level (include electron spin, but ignore spin of the proton)? The presence of a magnetic field $\vec{B} = B_0\hat{z}$ results in an additional term

$$H_B = \frac{eB}{2mc}(L_z + 2S_z)\,,$$

in the Hamiltonian. In the weak field limit, how do the energy of the states with $n = 3$ and their degeneracy affected?